Predicting shear flow transitions using machine-learning methods

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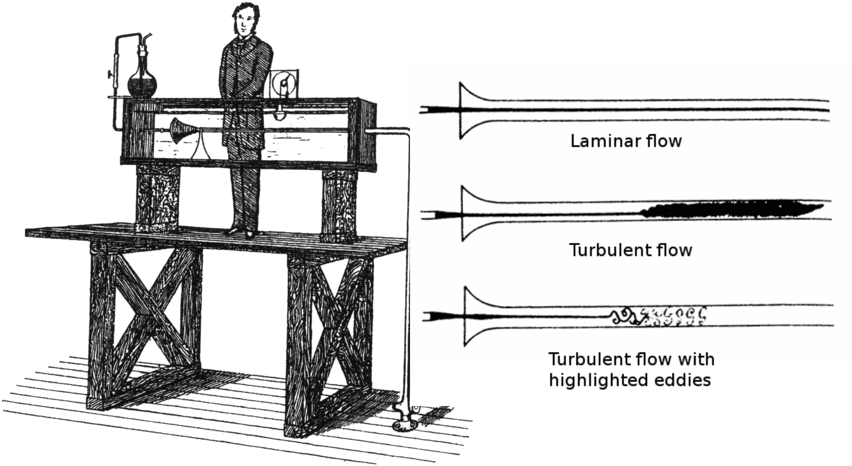
Reynolds experiment



Reynolds, Phil. Trans. R. Soc. London, 174 (1884)

Plane Couette flow

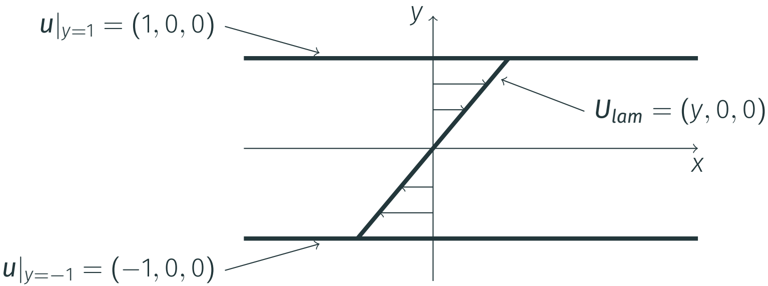
Incompressible Navier–Stokes equation:

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u}$$

$$\nabla \cdot \boldsymbol{u} = 0$$

Streamwise and spanwise directions: periodic BCs

Wall-normal direction: no-slip BCs



Moehlis–Faisst–Eckhardt model[1]

Low-dimensional model is obtained by Galerkin projection:

$$\boldsymbol{u}(\boldsymbol{x}, t) = \sum_{j=1}^{9} a_j(t)\boldsymbol{u}_j(\boldsymbol{x}).$$

9-dimensional system of ODEs:

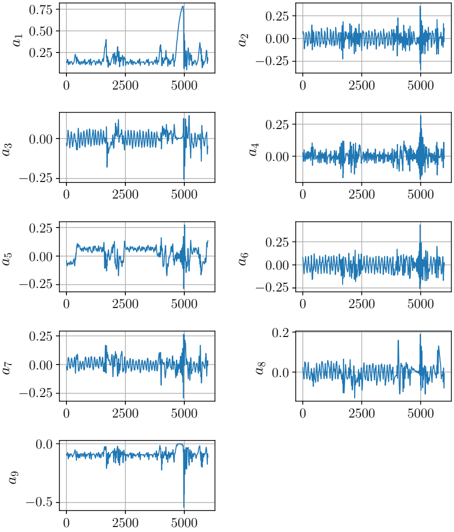
$$\frac{d}{dt}\boldsymbol{a} = \boldsymbol{f}(\boldsymbol{a}; Re, \Gamma_x, \Gamma_z),$$

where $\boldsymbol{a}(t) = [a_1(t), \ldots, a_9(t)]^T$.

Parameters:

- Domain wavelengths: $\Gamma_x = 1.75\pi, \Gamma_z = 1.2\pi$
- Reynolds number: $Re \in [200; 500]$

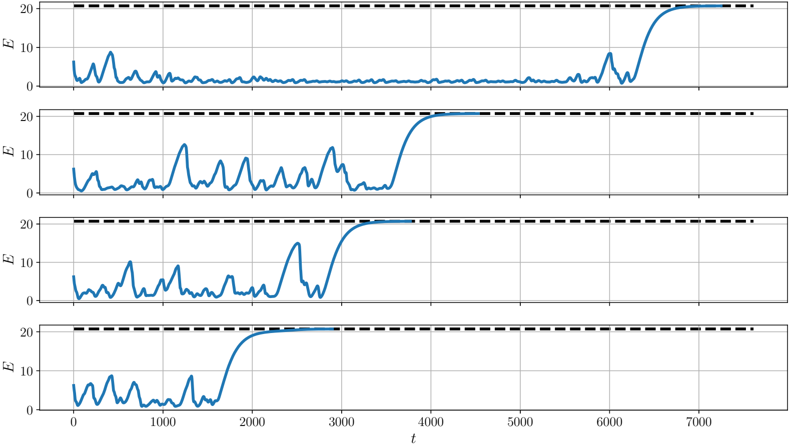
Sustained turbulence: $Re \gtrsim 320$



[1] Moehlis *et al.*, New J. Phys., **6** 56 (2004)

Laminarization ($Re = 300$)

- Turbulence in shear flows is a "leaky" attractor[2]
- As a result, all trajectories eventually end up with laminarization



[2]Avila *et al.*, Science **333**, 6039 (2011)

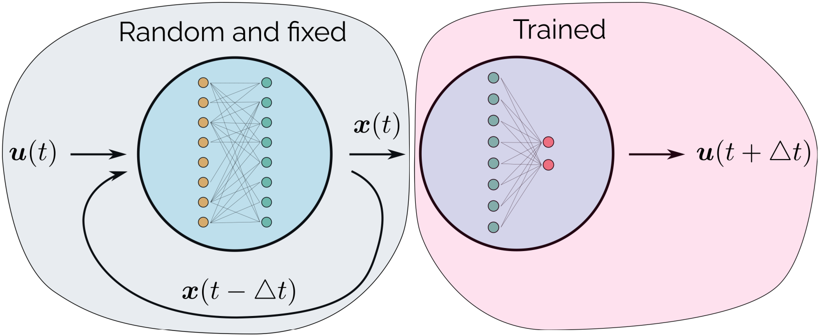
Echo State Network (ESN)

Echo State Network is a reservoir-computing architecture:

$$x(t) = \tanh(W_{in}u(t) + Wx(t - \triangle t)),$$
$$u(t + \triangle t) = W_{out}x(t).$$

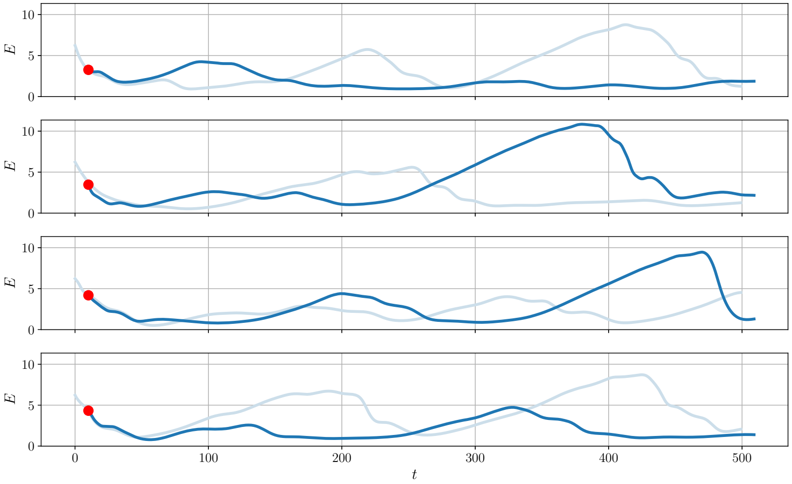
where

- W_{in} and W are random sparse matrices
- W_{out} is to be trained by solving the normal equation



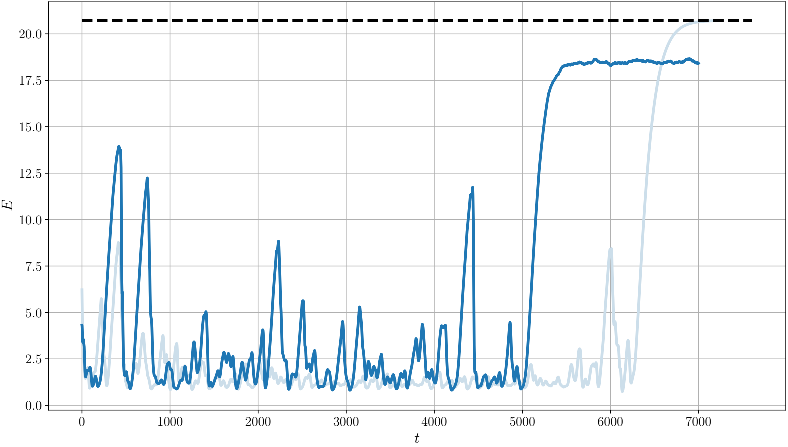
Short-term prediction ($Re = 300$)

Due to the chaotic nature of the original model, the ESN skill for short-term prediction is limited



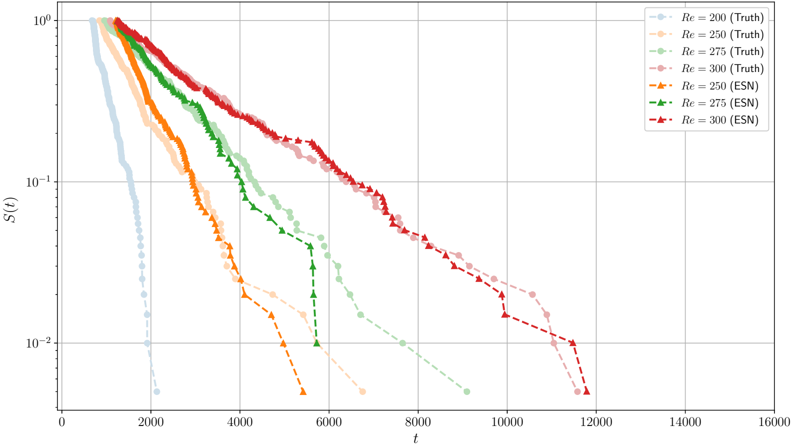
Long-term prediction ($Re = 300$)

- ESN is able to “learn” the laminarization dynamics without experiencing laminarization during the training
- Moreover, ESN is able to replicate the laminar solution



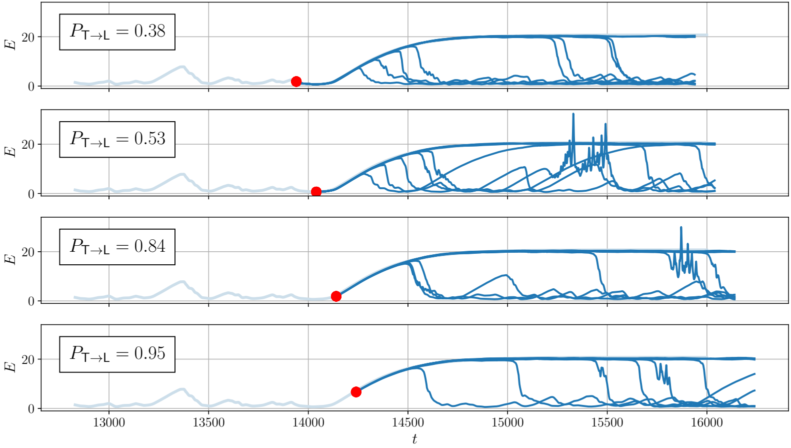
Lifetime distribution

- ESN can successfully replicate the lifetime statistics
- Its skill may degrade depending on the time series used for training



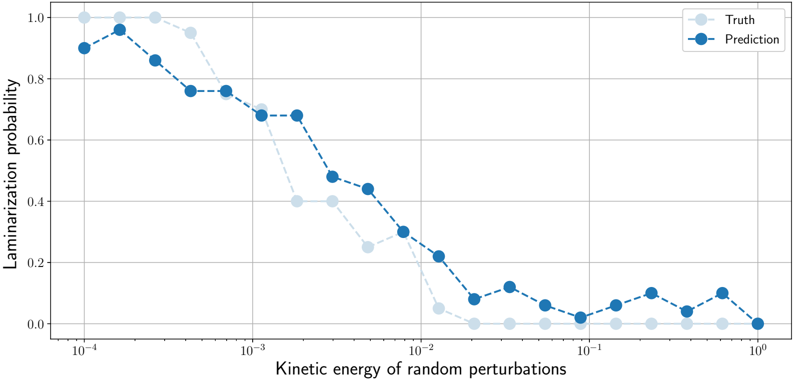
Turbulent-to-laminar transition ($Re = 500$)

- Ensemble approach can be used to estimate the probability of turbulent-to-laminar transition
- The probability grows as the initial condition gets closer to the laminarization event



Laminar-to-turbulent transition ($Re = 500$)

- Robustness of the laminar state to finite-amplitude perturbations is important for assessing laminar-to-turbulent transition
- **Laminarization probability** is the probability that a random perturbation decays as a function of its kinetic energy[3]
- ESN can successfully replicate the laminarization probability



[3] Pershin, Beaume, Tobias, J. Fluid Mech. **895**, A16 (2020)

Conclusion

PP2 Poster Session II (Wednesday, 9:30am):
Assessing the Control of Finite-Amplitude Instabilities
via a Probabilistic Protocol: Application to Transitional Flows
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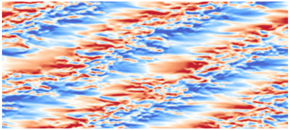
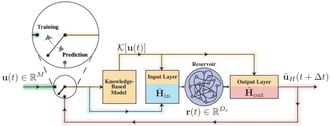
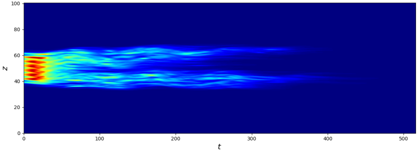
(a) **Spatially extended flows?**[4]



(b) **Physics-informed ESN?**[5]



(c) **Optimal control using reservoir computing?**



[4] Chantry *et al.*, J. Fluid Mech. **791**, R8 (2016)
[5] Pathak *et al.*, Chaos **28**, 041101 (2018)