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Dynamics of exact localized states in plane Couette flow

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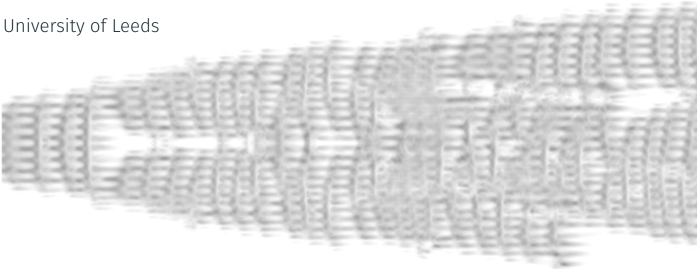
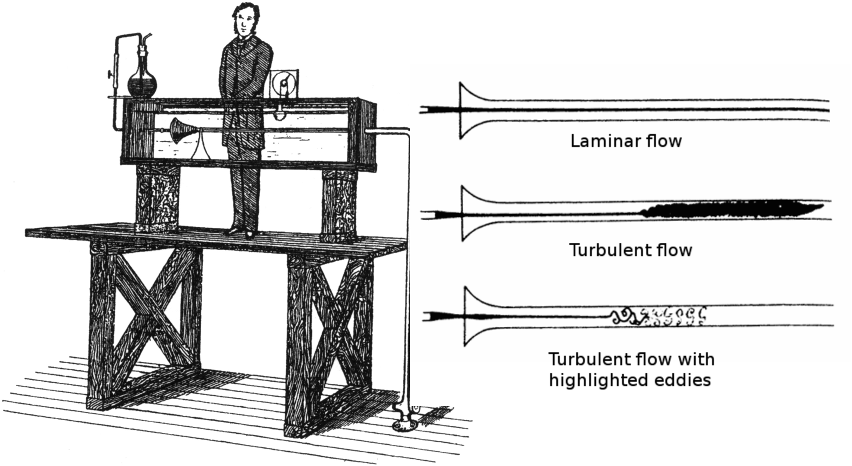




Reynolds, Phil. Trans. R. Soc. London, 174 (1884)

Plane Couette flow

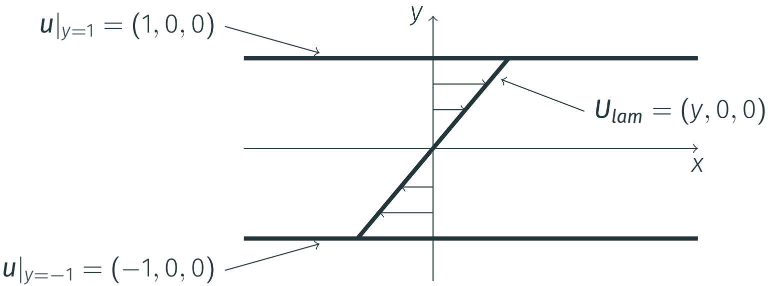
Incompressible Navier–Stokes equation:

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u}$$

$$\nabla \cdot \boldsymbol{u} = 0$$

Streamwise and spanwise directions: periodic BCs

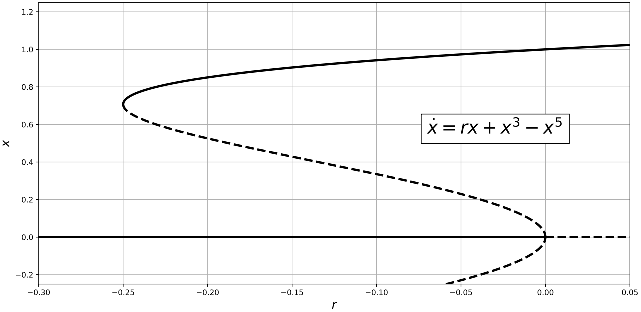
Wall-normal direction: no-slip BCs



Subcritical transitional flows

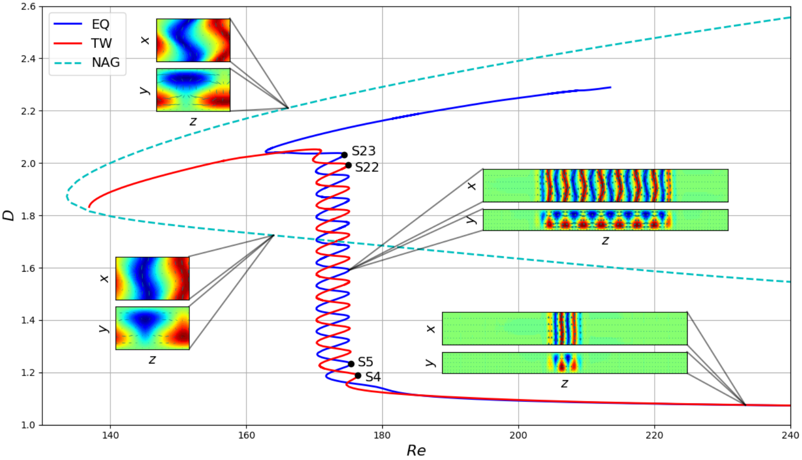
	Linearly stable laminar state	Sustained turbulence
Plane Couette flow	all Re	$Re \gtrsim 325$
Pipe flow	all Re	$Re \gtrsim 2040$
Plane Poiseuille flow	$Re \lesssim 5772$	$Re \gtrsim 840$

Transition is complicated by the coexistence of two attractive states:



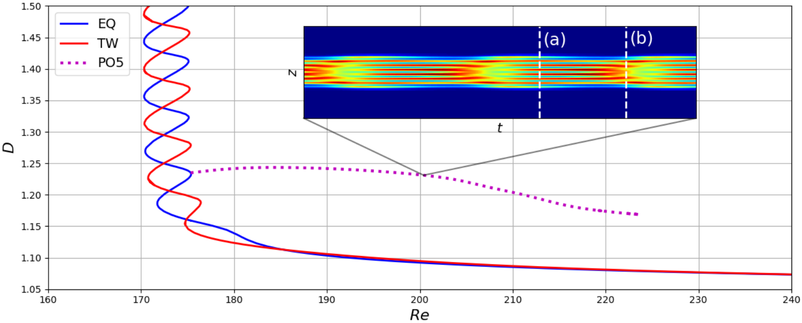
Snaking in plane Couette flow ($4\pi \times 2 \times 32\pi$)

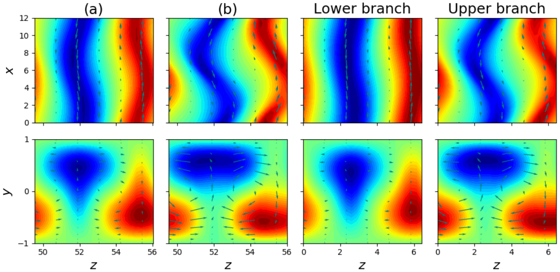
- First observed by Schneider *et al.* in 2010[1]
- Homoclinic snaking is most studied for the Swift–Hohenberg equation[2]



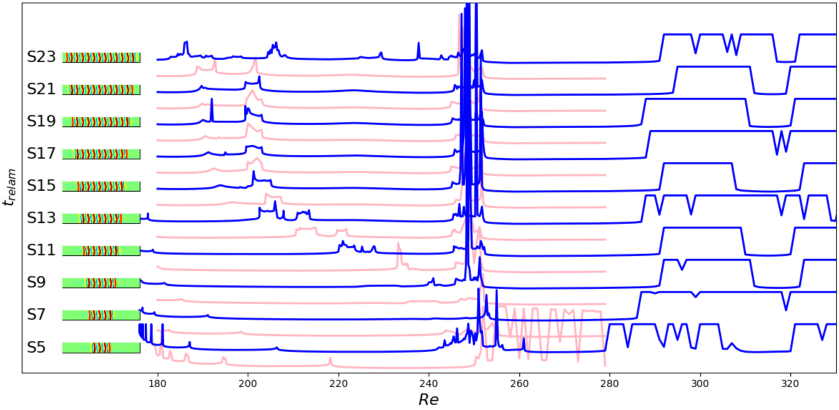
[1]Schneider *et al.*, Phys. Rev. Lett., **104** (2010)
[2]Knobloch, Annu. Rev. Condens. Matter Phys., **6** (2015)

Oscillatory dynamics ($Re \approx 200$)





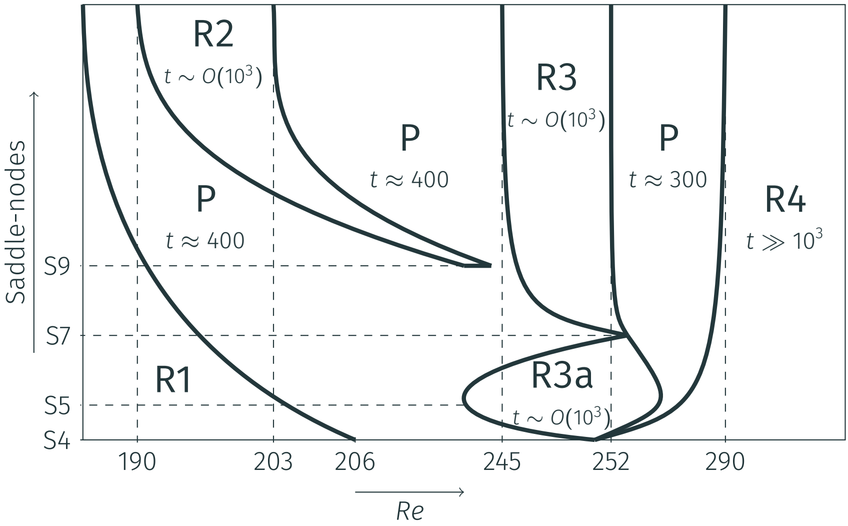
Relaminarisation times for localized states



Relaminarisation times for EQ (blue) and TW (red) saddle-node states. Midplane of streamwise velocity of EQ saddle-node states is shown on the left.

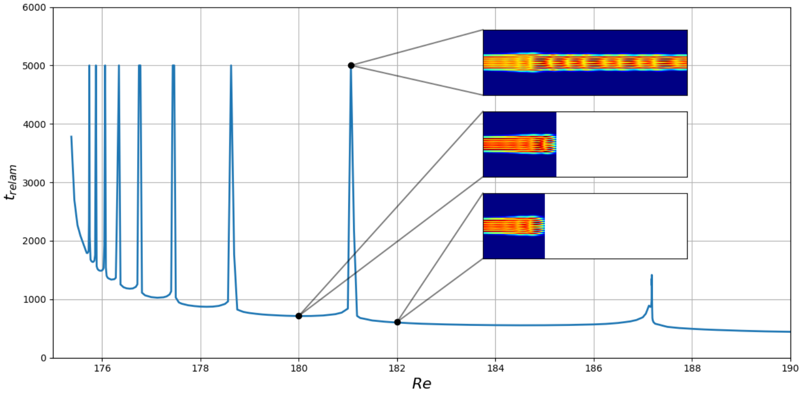
No major difference between the dynamics of EQ and TW

Map of the dynamics



- R1 peaks accumulating at Re_s are present for all initial states.
- Only wide enough states contain R2 and R3.

Region R1 – peaks (S5)

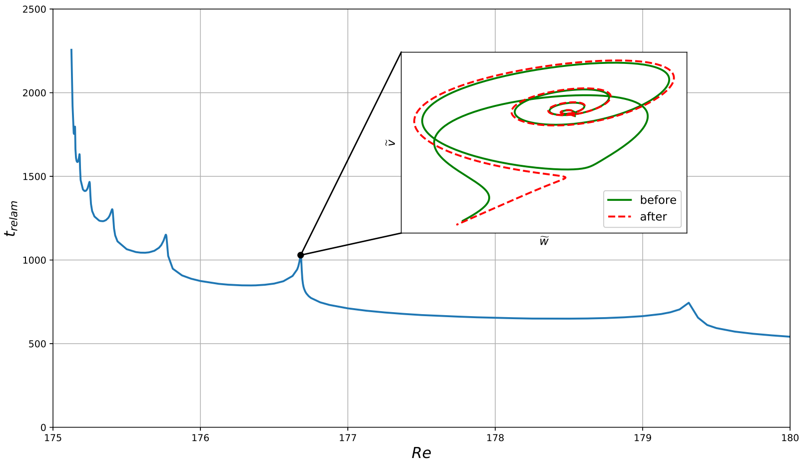


- Peaks: $Re_{n+1} - Re_s = \alpha\,(Re_n - Re_s)$
- Local minima: $t_n = t_0 + \beta n$

$$\Longrightarrow \quad t_{relam} = \frac{\beta}{\ln\alpha} \ln\left[\frac{2\,(Re - Re_s)}{(1+\alpha)\,(Re_0 - Re_s)}\right] + t_0$$

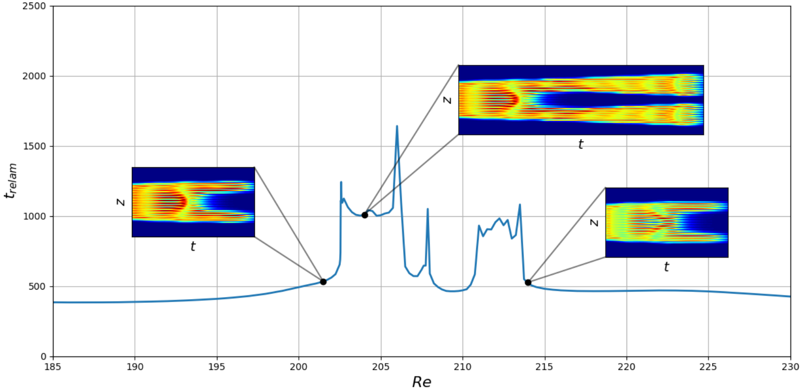
Region R1 – peaks (S7)

- For wider initial conditions, peaks are smooth
- Crossing a peak corresponds to the gain of one period



Region R2 – splitting

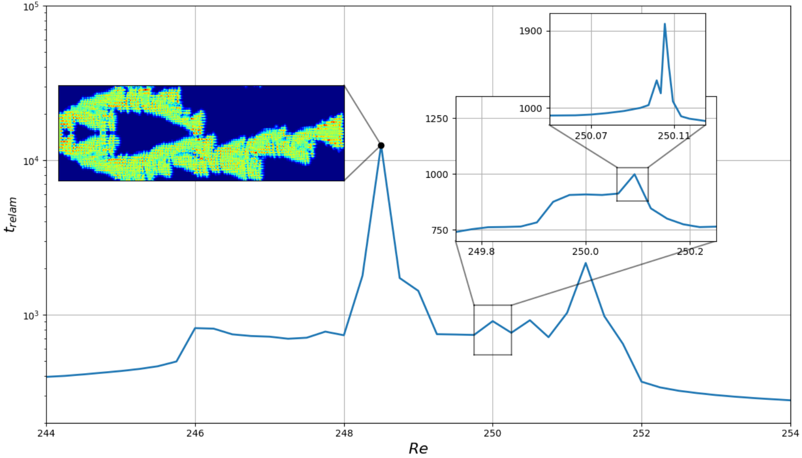
- Region R2 appears due to the creation and activation of spots
- The spot size is the same for all considered initial conditions



Relaminarisation times for S13 integrated for $Re \in [185; 230]$.

Region R3 – chaotic transients

- Like R2, R3 originates from the splitting of the initial spot
- Unlike R2, R3 contains long-lasting chaotic transients ($T > 4000$)



Relaminarisation times for S9 integrated for $Re \in [244; 254]$.

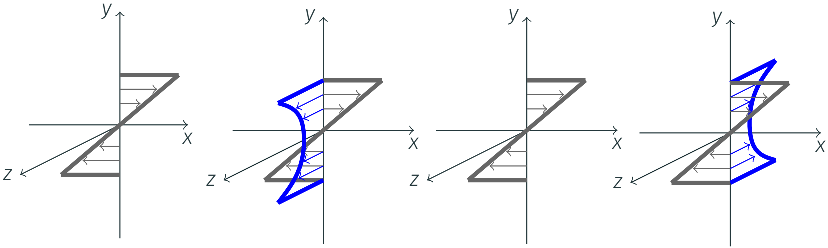
Control strategy: wall oscillations

We impose in-phase oscillations on the walls[3]:

$$u(x, \pm 1, z, t) = \pm \boldsymbol{e}_x + A sin(\omega t)\boldsymbol{e}_z$$

$$\implies \quad \boldsymbol{U}_{lam} = y\boldsymbol{e}_x + W(y,t)\boldsymbol{e}_z.$$

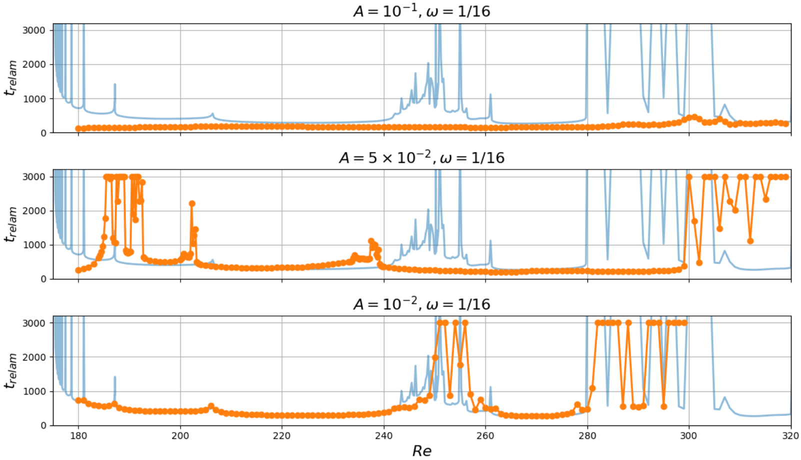
(a) $t = 0$: (b) $t = \frac{\pi}{2\omega}$: (c) $t = \frac{\pi}{\omega}$: (d) $t = \frac{3\pi}{2\omega}$:



[3]Motivated by Rabin *et al.*, J. Fluid Mech. **738** (2014)

Homotopy from the uncontrolled case for S5

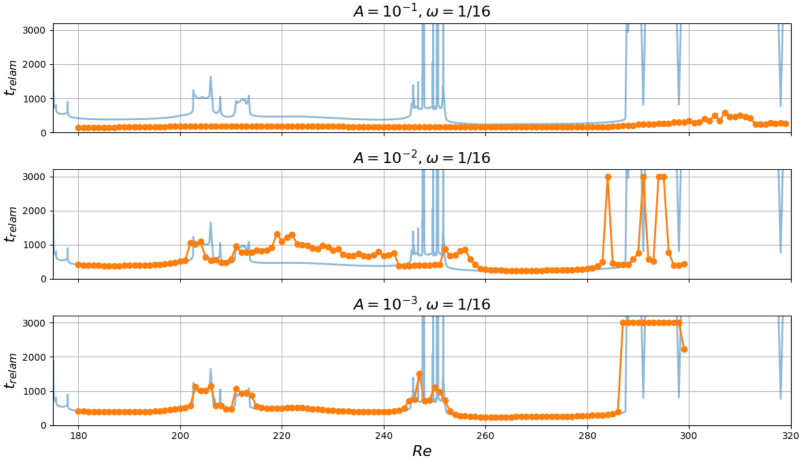
- Fast relaminarization for $A \sim O(10^{-1})$
- Original regions are recovered for $A \lesssim 10^{-2}$



Relaminarisation times for the uncontrolled (blue) and wall-oscillated (orange) cases.

Homotopy from the uncontrolled case for S13

- Fast relaminarization for $A \sim O(10^{-1})$
- Original regions are recovered for $A \lesssim 10^{-3}$



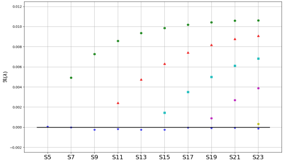
Relaminarisation times for the uncontrolled (blue) and wall-oscillated (orange) cases.

Conclusion

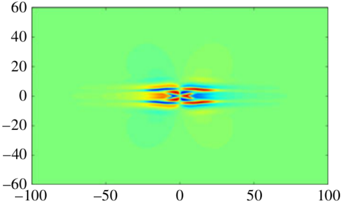
Details: Pershin, Beaume and Tobias, *J. Fluid Mech.* **867**, 414–437 (2019)

(a) **Stability analysis of the snakes?**

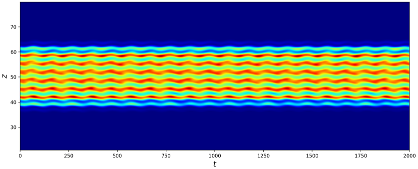
comparison with Beaume, *et al.*, J. Fluid Mech., 840 (2018)



(b) **Doubly localized solutions?**[4]



(c) **Dynamics in wall-oscillated plane Couette flow?**



[4] Brand and Gibson, J. Fluid Mech. **750**, R3 (2014)