Relaminarisation of exact localised states in transitional plane Couette flow

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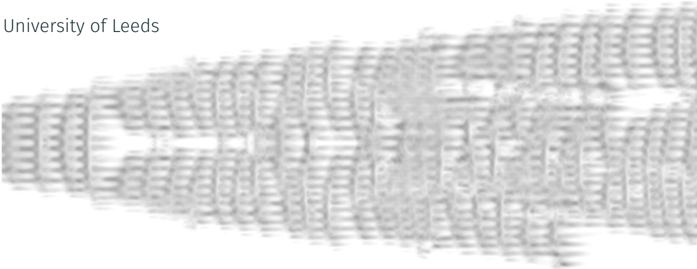
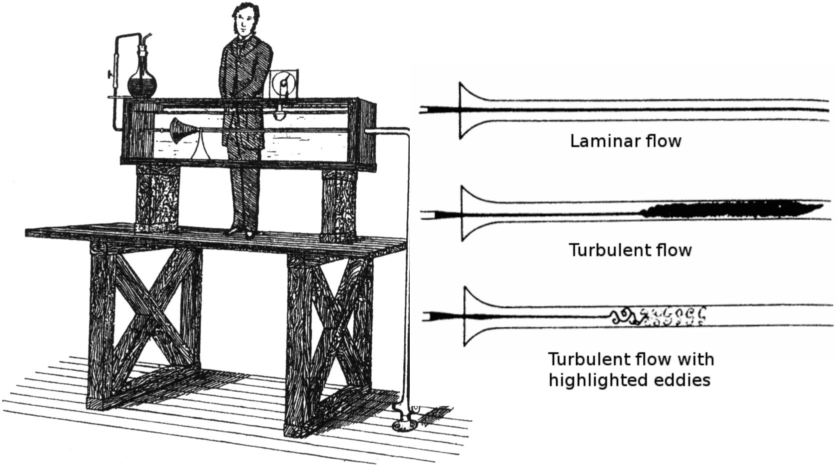


Reynolds experiment



Reynolds, Phil. Trans. R. Soc. London, 174 (1884)

Questions and approaches

Questions:

- Re_g for stability of laminar state?
- Re_c for stability of turbulent state?
- If $Re_g \neq Re_c$, what determines whether the flow goes laminar or turbulent in between?

Statistical approach:

- spots splitting vs. decay probabilities
  - e.g., Avila *et al.*, Science 333, 192 (2011)
- directed percolation
  - e.g., Chantry *et al.*, J. Fluid Mech. 824, R1 (2017)

Dynamical systems approach:

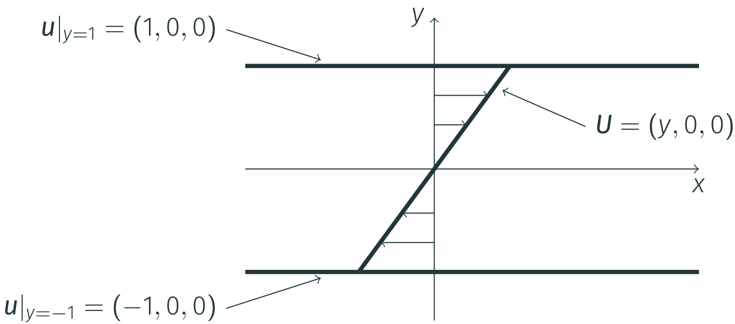
- bifurcations to chaos
- invariant solutions as a skeleton of a chaotic attractor
  - e.g., Kawahara *et al.*, Annu. Rev. Fluid Mech. 44, 203 (2012)

Navier–Stokes equation:

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u}$$

Incompressibility condition: $\nabla \cdot \boldsymbol{u} = 0$

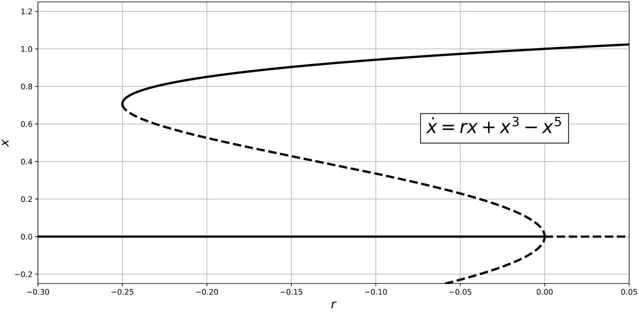
Domain: $\mathbb{R} \times [-1; 1] \times \mathbb{R}$



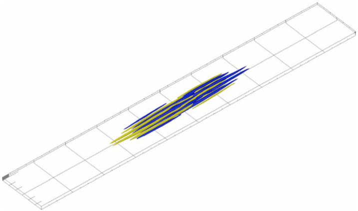
Subcritical transitional flows

	Linearly stable laminar state	Sustained turbulence
Plane Couette flow	all *Re*	$Re \gtrsim 325$
Pipe flow	all *Re*	$Re \gtrsim 2040$
Plane Poiseuille flow	$Re \lesssim 5772$	$Re \gtrsim 840$

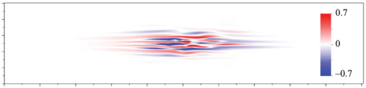
Transition is complicated by the coexistence of two attractive states:



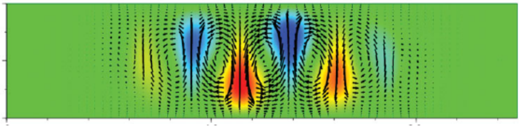
(a) $63.7\pi \times 2 \times 15.9\pi$ (Duguet *et al.*, Phys. Fluids, 21, 111701 (2009)):



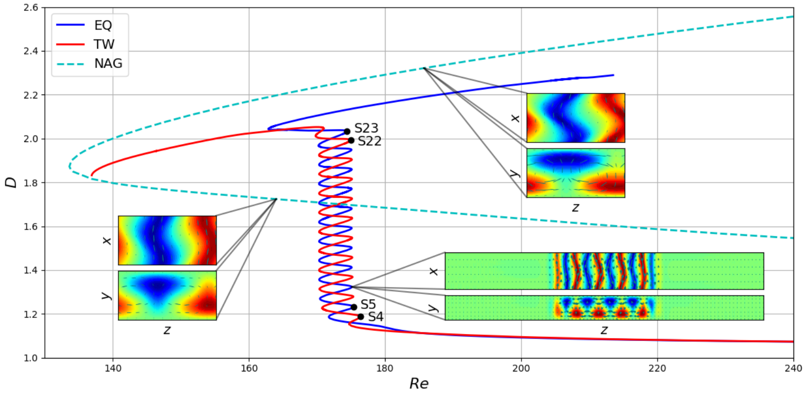
(b) $64\pi \times 2 \times 16\pi$ (Schneider, *et al.*, J. Fluid Mech., 646 (2010)):



(c) $4\pi \times 2 \times 8\pi$ (Schneider, *et al.*, J. Fluid Mech., 646 (2010)):



Snaking in plane Couette flow ($4\pi \times 2 \times 32\pi$)

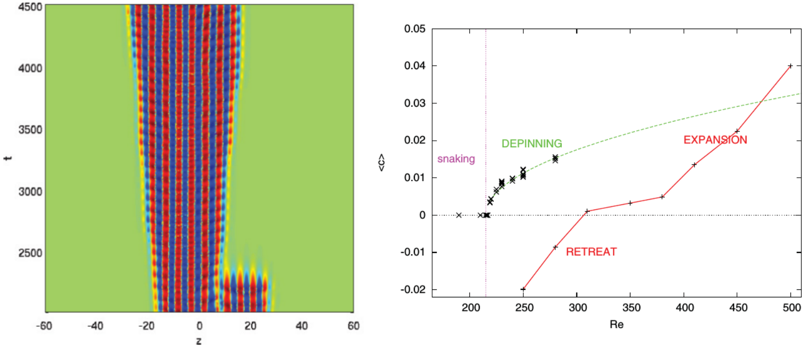


First observed by Schneider *et al.*, Phys. Rev. Lett., 104 (2010).

Model of homoclinic snaking is provided by Swift–Hohenberg equation (Knobloch, Annu. Rev. Condens. Matter Phys., 6 (2015))

Depinning: expanding/collapsing of the initial spatial pattern outside the snaking by nucleation/annihilation of cells.

It is characterised by a square-root law of the speed of front: $c \propto |Re - Re_s|^{1/2}$.



Evidence of depinning (Duguet *et al.*, Phys. Rev. E, 84 (2011))

Transition problem

1. Does depinning describe transition to turbulence?
2. If not, how do localised states turn to turbulence?

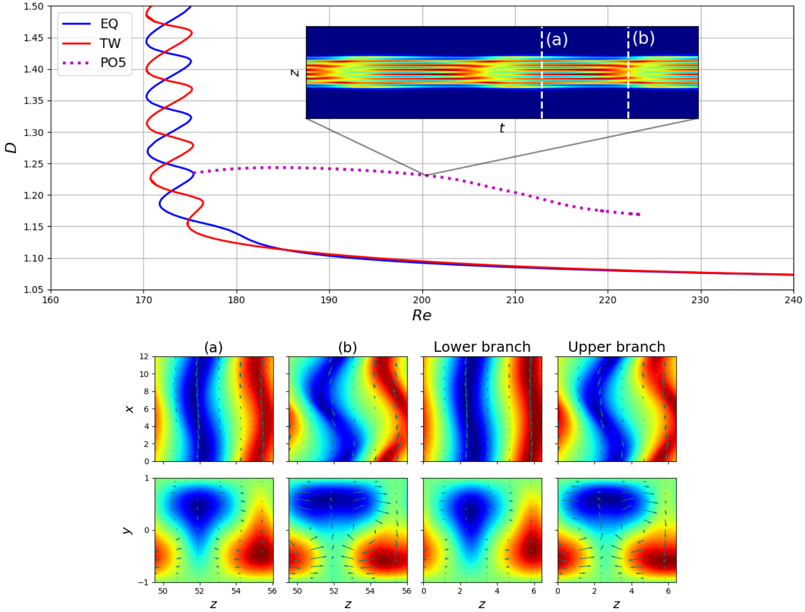
Let's consider the time-evolution of the right saddle-nodes states for $Re \in (Re_s; 350]$.

The problem set-up:

- Large spanwisely extended domain: $4\pi \times 2 \times 32\pi$.
- Initial conditions: right saddle-nodes states, $Re \in (Re_s; 350]$.
- Pseudo-spectral solver with Fourier–Chebyshev–Fourier transform at $32 \times 33 \times 512$ collocation knots.
- 1.6×10^6 degrees of freedom.

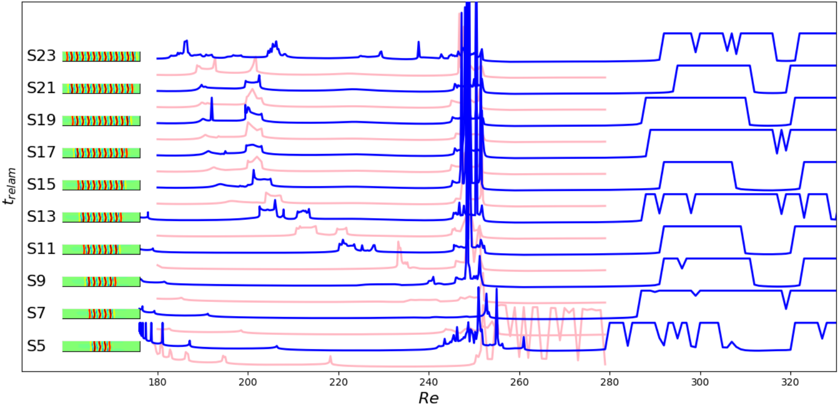
Code: channelflow (John Gibson).

Oscillatory dynamics ($Re \approx 200$)



Relaminarisation times for localised states

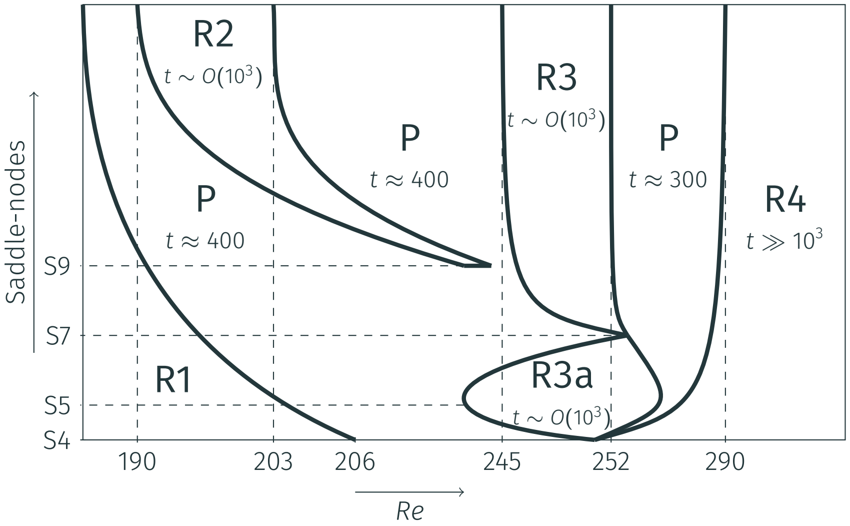
No depinning!



Relaminarisation times for EQ and TW saddle-nodes states (blue and red curves resp.). Midplane of streamwise velocity of EQ saddle-nodes is shown on the left.

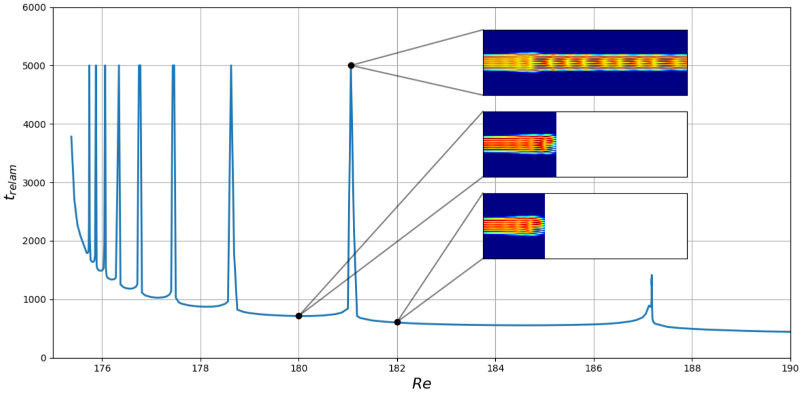
No principal difference between the dynamics of EQ and TW.

Map of the dynamics



- R1 peaks accumulating at Re_s are present for all initial states.
- Only wide enough states contain R2 and R3.

Region R1 – peaks (S5)

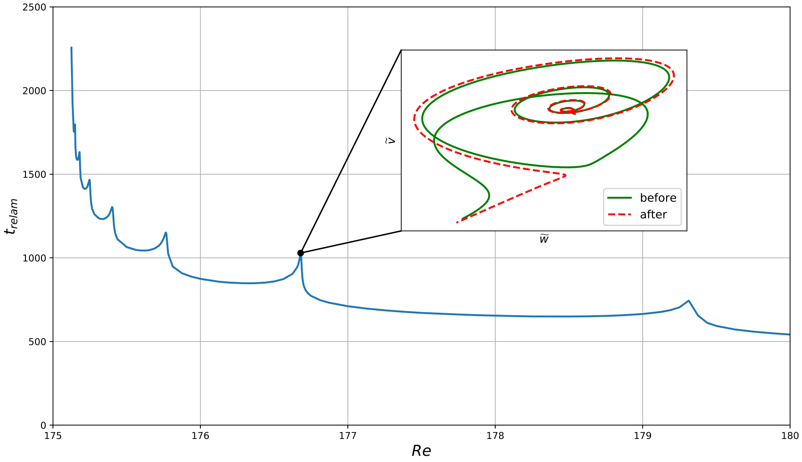


- Peaks: $Re_{n+1} - Re_s = \alpha\,(Re_n - Re_s)$
- Local minima: $t_n = t_0 + \beta n$

$$\Longrightarrow \quad t_{relam} = \frac{\beta}{\ln \alpha} \ln \left[\frac{2\,(Re - Re_s)}{(1+\alpha)\,(Re_0 - Re_s)} \right] + t_0$$

Region R1 – peaks (S7)

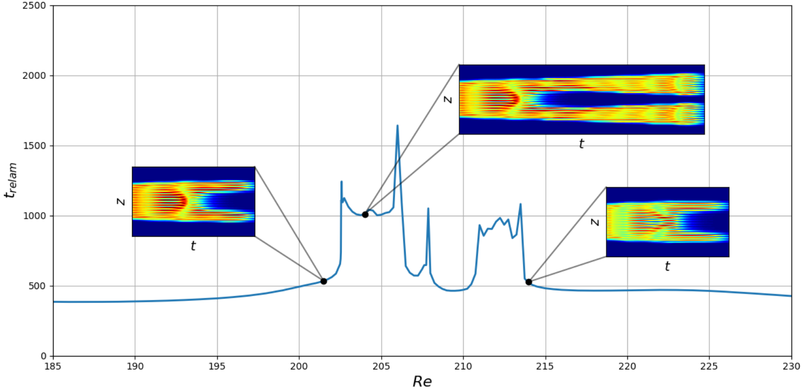
For wider initial conditions, peaks are smoothed.



Crossing a peak corresponds to a loss of a cycle.

Region R2 – splitting

Region R2 appears due to the creation of the spots and their activation.



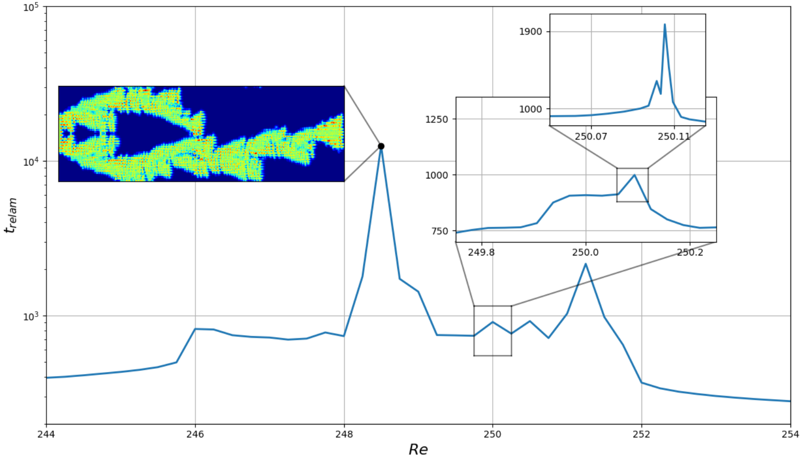
Relaminarisation times for S13 integrated for $Re \in [185; 230]$.

The size of spots is the same for all considered initial conditions.

Region R3 – chaotic transients

Like R2, R3 originates from the splitting of the initial spot.

Unlike R2, R3 contains long-lasting chaotic transients ($T > 4000$).



Relaminarisation times for S9 integrated for $Re \in [244; 254]$.

Conclusion

Future work:

- Stability analysis of the snaking branches
  - $\Longrightarrow$ comparison with Beaume, *et al.*, J. Fluid Mech., 840 (2018)
- Control of relaminarisation times
- Control of front speed in the transitional regime

Open questions:

- What determines the presence of depinning?
- How could we characterise the dynamics inside R2 and R3 apart from simple statistics?